Compensation for the effects of ambient conditions on the calibration of multi-capillary pressure drop standards

by

Stéphane Colard[(1)], Wolfgang Trinkies[(2)], Georges Cholet[(3)], Barry Camm[(4)], Malcolm Austin[(5)], Rodolphe Gualandris[(6)]

[(1)]ALTADIS Research Centre, 4 rue André Dessaux, 45404 Fleury-Les-Aubrais, France
[(2)]BAT, Bayreuth, Germany
[(3)]Sodim Instrumentation, Fleury-Les Aubrais, France
[(4)]Borgwaldt-kc, Hamburg, Germany
[(5)]Cerulean, Milton-Keynes, UK
[(6)]Philip Morris International, Neuchâtel, Switzerland

Summary :

Cigarette draw resistance and filter pressure drop are both major physical parameters for the tobacco industry. Therefore these parameters must be measured reliably. For these measurements, specific equipment is used which is calibrated with pressure drop transfer standards. Each transfer standard must have A known and stable pressure drop value, such standards usually being composed of several capillary tubes associated in parallel. However, pressure drop values are modified by the ambient conditions during the calibration, i.e. by the temperature and relative humidity of air, and the atmospheric pressure. In order to reduce the influence of these ambient factors, a physical model was developed for compensating for the effects of ambient conditions on the calibration of multi-capillary pressure drop standards.

Experiments demonstrated that the standards exhibited a turbulent airflow component, which explains why atmospheric pressure effects have an effect on the calibrated value. The standards were also found to show a high degree of sensitivity to the ambient temperature, but low sensitivity to relative humidity.

The developed compensation model has been implemented in a spreadsheet facilitating its use, and has been applied successfully to calibration results with wide ranging ambient conditions. Finally, to simplify the process of compensation, a simple equivalent mathematical model was developed.

In conclusion, the results of this study demonstrate the benefits to calibration data of minimising the effects of ambient conditions.

* corresponding author : stephane.colard@altadis.com

1. INTRODUCTION

Cigarette draw resistance and filter pressure drop are both major physical parameters for the tobacco industry, and it is important that they are measured reliably. For these measurements, equipment calibrated with pressure drop (PD) transfer standards is used. PD standards are normally composed of ten capillary tubes associated in parallel, the structure being made of glass. Each standard must have a known and stable value ascribed to it. However, PD values are influenced by the ambient conditions during calibration. One way of reducing the influence of these ambient factors is the derivation and use of a compensation formula. In the framework of PD standard calibration, the objective of the compensation formula is the calculation of a PD value, PD_S, at standard ambient conditions as defined in ISO 6565, CORESTA RM41 and ISO 3402 (T_S = 22°C, H_S = 60%, P_S = 1013 hPa, Outlet airflow Q_S = 17.5ml/s), from PD measurements undertaken at different conditions (T,H,P,Q).

This work has been done by the Coresta Task Force "Calibration of PD Transfer Standards" and only applies to the most commonly used standards comprised of 10 glass capillary tubes.

2. THEORY

The ambient conditions modify two parameters that influence the airflow behaviour: the viscosity η and the density ρ of air. Rasmussen [1] has developed a calculation procedure for these two parameters from which, by fitting (R^2=99.9%) we deduced two simplified formulae covering wide ranges of ambient conditions generated in the participating laboratories [18-26°C], [50-70%], [900-1100hPa]:

$$\eta(T,H)(Pa.s) = 4.703\times10^{-6} + 4.587\times10^{-8}\times T(K) - 4.944\times10^{-10}\times H(\%)$$

$$\rho(P,T)(kg/m^3) = 2.032\times10^{-1} - 7.137\times10^{-4}\times T(K) + 2.281\times10^{-5}\times P(Pa) - 3.728\times10^{-8}\times T(K)\times P(Pa)$$

2.1 Laminar Airflow

Historically PD standards composed of several capillary tubes have been used, because it is presumed that the airflow through them is laminar, corresponding to a low Reynolds number (Re<2000). In this case, the PD is proportional to the air viscosity and to the volumetric airflow. In accordance with the perfect gas law it can be written :

$$\frac{PD_S}{PD} = \frac{\eta(T_S,H_S)\times Q_S(P_S,T_S,PD_S)}{\eta(T,H)\times Q(P,T,PD)} = \frac{\eta(T_S,H_S)\times T_S\times(P-PD)}{\eta(T,H)\times T\times(P_S-PD_S)}$$

The development of the previous formula gives a second order polynomial expression that is easy to solve :

$$PD_S^2 - P_S \times PD_S + \frac{\eta_S \times T_S}{\eta \times T} \times (P - PD) \times PD = 0$$

The standard PD value with an outlet volumetric airflow equal to 17.5ml/s is then :

$$PD_{S,17.5ml/s} = PD_S \times \frac{17.5}{Q(P_S, T_S, PD_S)}$$

2.2 Turbulent Airflow

If a totally turbulent airflow is considered, the PD value is proportional to the air density and to the square of the volumetric airflow. In accordance with the perfect gas law it can be written:

$$\frac{PD_S}{PD} = \frac{\rho(P_S, T_S) \times Q_S^2(P_S, T_S, PD_S)}{\rho(P, T) \times Q^2(P, T, PD)} = \frac{\rho(P_S, T_S)}{\rho(P, T)} \times \left(\frac{T_S \times (P - PD)}{T \times (P_S - PD_S)} \right)^2$$

The development of the previous formula gives a third order polynomial expression:

$$PD_S^3 - 2 \times P_S \times PD_S^2 + P_S^2 \times PD_S - \frac{\rho_S \times T_S^2}{\rho \times T^2} \times PD \times (P - PD)^2 = 0$$

The standard PD value with an outlet volumetric airflow equal to 17.5ml/s is then:

$$PD_{S,17.5ml/s} = PD_S \times \left(\frac{17.5}{Q(P_S, T_S, PD_S)} \right)^2$$

2.3 Laminar + Turbulent Airflow

In order to consider the case of an airflow that is partly laminar and partly turbulent, it is necessary to introduce a coefficient of turbulence x (%) such as :

$$PD_1 = x \times PD$$

$$PD_2 = (100 - x) \times PD$$

where PD_1 and PD_2 are the pressure drop observed across the turbulent part and across the laminar part respectively. It is assumed here that the turbulence is mainly observed at the inlet of the standard.

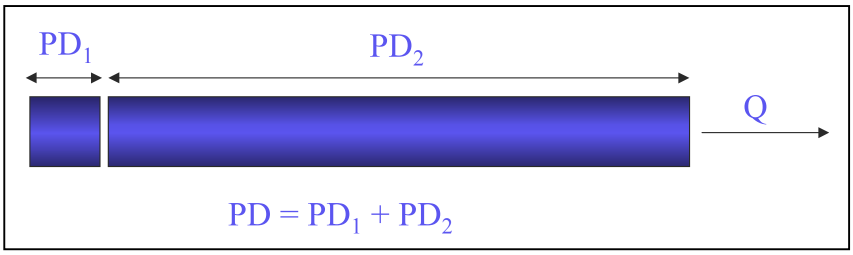

Figure 1 – Composition of the PD components across a standard

In accordance with part 2.1, the laminar component may be expressed by the following equation where PD_{1S} and PD_{2S} are the unknown parameters :

$$PD_{2S}^{2}-\left(P_{S}-PD_{1S}\right)\times PD_{2S}+\frac{\eta_{S}\times T_{S}}{\eta\times T}\times\left(P-PD\right)\times PD_{2}=0$$

In accordance with part 2.2, the turbulent component can be expressed by :

$$PD_{1S}^{3}-2\times P_{S}\times PD_{1S}^{2}+P_{S}^{2}\times PD_{1S}-\frac{\rho_{S}\times T_{S}^{2}}{\rho\times T^{2}}\times PD_{1}\times\left(P-PD_{1}\right)^{2}=0$$

where PD_{1S} is the unknown parameter.

Therefore, in the case of a laminar and turbulent airflow, two equations were obtained having two unknown parameters, PD_{1S} and PD_{2S}. After resolving these equations, using for example an iterative Newtonian method, the standard PD value with an outlet volumetric airflow of 17.5ml/s is then given by:

$$PD_{S,17.5ml/s}=PD_{1S}\times\left(\frac{17.5}{Q\left(P_{S},T_{S},PD_{S}\right)}\right)^{2}+PD_{2S}\times\left(\frac{17.5}{Q\left(P_{S},T_{S},PD_{S}\right)}\right)$$

The compensation models previously developed have been introduced in a spreadsheet for facilitating its use.

3. EXPERIMENTS

3.1 Compensation for atmospheric pressure effects - Level of turbulence *x*

Experiments show very low non-linear behaviour for the relationship of PD values versus airflow for multi-capillary standards. This excludes the hypothesis of a totally turbulent airflow. However, experiments also show that the PD measurement is affected by the atmospheric pressure (see figure 2). This observation means that the air density modifies the PD and the hypothesis of a totally laminar airflow can also be excluded. Therefore, it can be concluded that the airflow through a multi-capillary PD standard is partly laminar, and partly turbulent (§2.3). This assumption is in agreement with the results obtained by Keith [2].

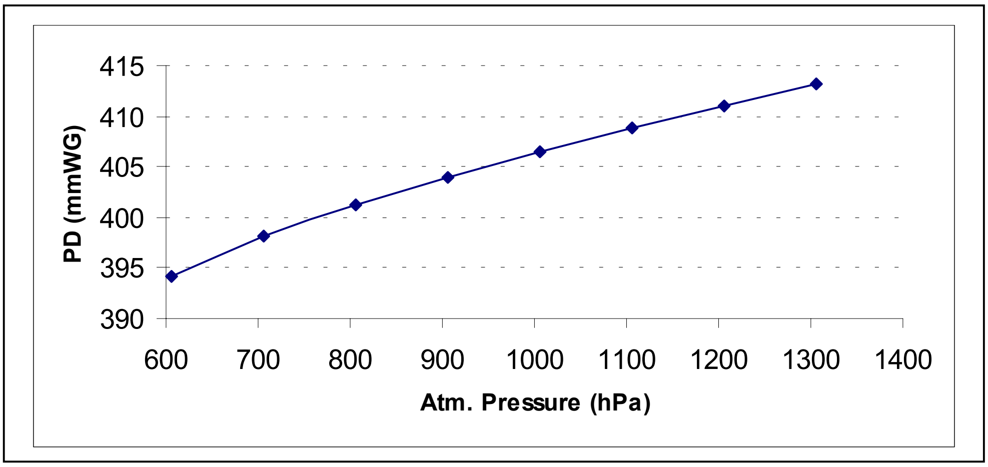

Figure 2 – PD versus atmospheric pressure (Sodim) [rH and T are kept constant]

The level of turbulence *x* which allowed the best compensation, i.e. a minimal standard deviation, was determined from measurements of PD versus atmospheric pressure over the range [900-1100hPa]. The following results were obtained :

PD level (mmWG)	Standard deviation without compensation (mmWG)	Standard deviation with compensation (mmWG)	Level of turbulence *x* (%)
200	0.85	0.04	3.9
400	2.45	0.08	5.0
600	4.10	0.07	5.4
800	6.60	0.12	6.0

Table 1 – Levels of turbulence minimising the standard deviations of the compensated values

The increasing of *x* with increasing PD levels could be attributed to the decreasing diameters of the capillaries. This may induce a higher level of turbulence.

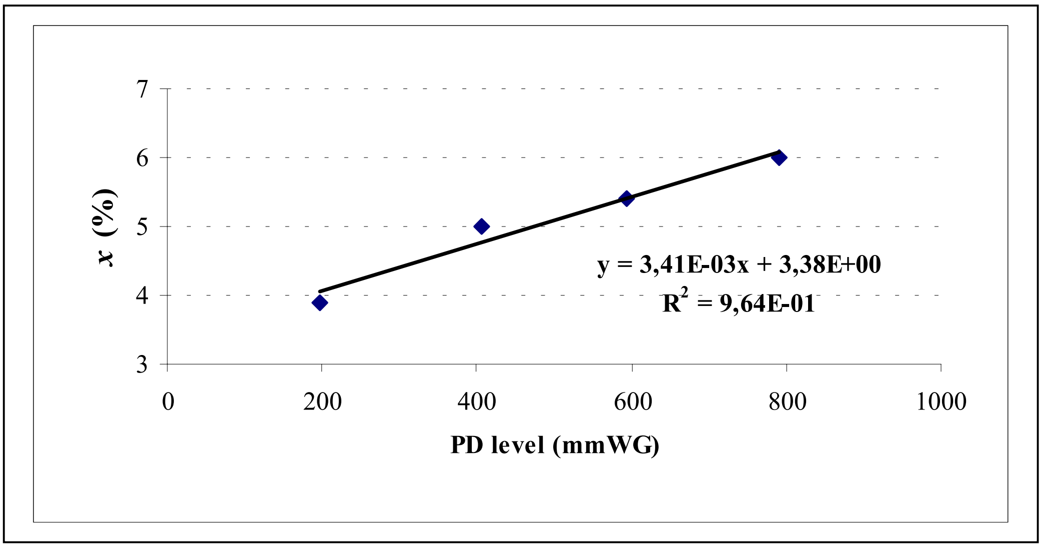

Figure 3 – Level of turbulence versus PD level

The compensation for the atmospheric pressure effects is illustrated on figure 4.

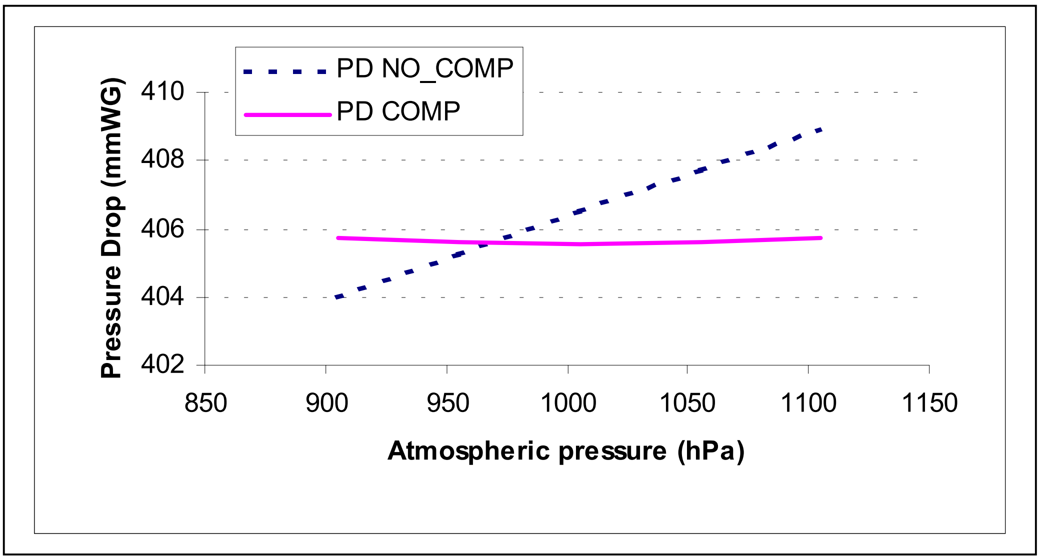

Figure 4 – Compensation for Atmospheric Pressure effects

Having established x, it is now possible to validate the compensation process using experimental data with widely varying ambient conditions. It is possible to calculate the theoretical sensitivity of the PD value to the ambient condition parameters. From the model, the theoretical sensitivity of PD to the atmospheric pressure varies from 0.22% to 0.41% of the PD value per 50hPa for 200 to 800mmWG PD levels respectively.

3.2 Compensation for ambient temperature effects

In order to evaluate compensation for the effects of temperature, the variation of the PD versus ambient temperature over the range [18-28°C] was measured. The level of turbulence was adjusted according to the PD level using the linear relation, drawn in figure 3. Compensation produced the following results:

PD level (mmWG)	Standard deviation without compensation (mmWG)	Standard deviation with compensation (mmWG)
200	1.31	0.11
400	2.61	0.36
600	3.81	0.54
800	4.43	0.91

Table 2 – Reduction of the Standard Deviation following compensation

Table 2 clearly shows that the compensation significantly decreased the standard deviation of the measured values. These results are illustrated graphically in figure 5 by the reduction of the absolute value of the slope of the curve.

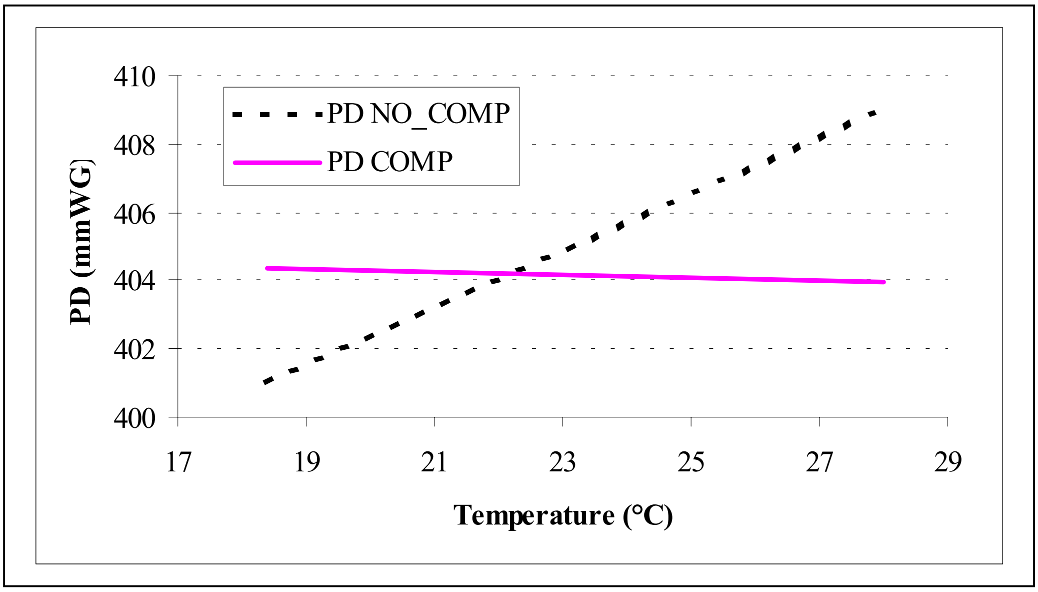

Figure 5 – Compensation for ambient temperature effects

The weak over-compensation which is observed (Fig. 5) may be due to the fact that the physical model developed in the part 2.3 doesn't take into account the temperature effect on the standard itself. From the model, the theoretical sensitivity of PD values to ambient temperature is about 0.23% of the PD value per degree Celsius.

3.3 Compensation for relative humidity effects

In order to evaluate compensation for the effects of relative humidity, the variation of the PD versus relative humidity over the range [25-80%] was measured. The level of turbulence was adjusted according to the PD level using the linear relation drawn in figure 3. Compensation produced the following results:

PD level (mmWG)	Standard deviation without compensation (mmWG)	Standard deviation with compensation (mmWG)
200	0.12	0.18
400	0.53	0.53
600	0.42	0.48
800	0.72	0.33

Table 3 – Reduction of the Standard Deviation following compensation

Table 3 only shows a decrease in the standard deviation for the highest PD level. The measured variation of the PD versus relative humidity seems to be so slight, that the compensation has no significant effect on the standard deviation. From the model developed in part 2.3, the calculated sensitivity of the PD to the relative humidity is approximately –0.003% of the PD value per %rH. The poor efficiency of the compensation may be explained by the low sensitivity of PD values to Relative Humidity when combined with the natural variation of the measurements.

3.4 Application to a long-term calibration

To complete the experiments, the compensation formula was applied to 29 calibration results measured in the same laboratory with two standards (400 and 800mmWG PD level) over a period of two months. During this period the temperature varied from 20.1°C to 23.9°C, the atmospheric pressure from 1000hPa to 1025hPa and the relative humidity from 58% to 64%.

PD level (mmWG)	Standard deviation without compensation (mmWG)	Standard deviation with compensation (mmWG)
400	1.26	0.66
800	2.42	1.29

Table 4 – Compensation results with PD standard calibrated over a period of time

By using the compensation formula, the standard deviation is approximately halved.

4. A SIMPLIFIED MATHEMATICAL FORMULA

In order to avoid an iterative resolution of the physical equations, an equivalent mathematical formula was developed for easier application. A curve for the relative variation of the PD value versus the PD value itself was fitted. We obtained :

$$\frac{\Delta PD}{PD_{Meas}}(\%) = \alpha = a_0 - \Delta T \times \left(a_1 + a_2 \times PD_{Meas}\right) - \Delta P_{atm} \times \left(a_3 + a_4 \times PD_{Meas}\right)$$
$$+ a_5 \times \Delta RH + a_6 \times \left(\Delta P_{atm}\right)^2 + a_7 \times PD_{Meas}$$

with difference ΔT between the ambient temperature and 22°C

difference ΔP_{atm} between the atmospheric pressure and 1013hPa

difference ΔRH between the relative humidity and 60%

$R^2 = 99.96\%$

$a_0 = -2.555 \times 10^{-2}; a_1 = 2.404 \times 10^{-1}; a_2 = -2.214 \times 10^{-5}; a_3 = 2.931 \times 10^{-3};$

$a_4 = 6.617 \times 10^{-6}; a_5 = 2.708 \times 10^{-3}; a_6 = 7.128 \times 10^{-6}; a_7 = 5.617 \times 10^{-5}$

The compensated PD value is calculated using the following formula :

$$PD_{S,17.5ml/s} = \left[PD_1 \times \left(\frac{17.5}{Q}\right)^2 + PD_2 \times \left(\frac{17.5}{Q}\right)\right] \times \left(1 + \frac{\alpha}{100}\right)$$

5. CONCLUSION

Experiments clearly show that the airflow through a multi-capillary standard is partly turbulent. A physical model was developed, that includes a parameter related to the level of turbulence varying from 3.9 to 6%. The use of this model allows successful compensation for the effects of atmospheric pressure and ambient temperature. The sensitivity of the pressure drop to the relative humidity is so low that compensation seems unnecessary for this parameter but is retained in the formula. Finally, in order to facilitate the compensation process, a simplified mathematical formula is proposed.

References :

[1] K. Rasmussen, Technical University of Denmark; Report PL-11b. (1997)

[2] C.H. Keith and J.A. Corbin, Beïtrage zur Tabakforschung, 8, 2 (1975), 60-64